Extrudate Swell of Newtonian Fluids from Annular Dies

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Introduction

Extrudate swell of Newtonian fluids has been studied extensively for flows through slit and capillary dies. Both experimental and numerical results have been obtained by several researchers. The first experimental results were reported for capillary dies (Middleman and Gavis, 1961). The first numerical results for capillary dies were obtained by Tanner (1973) who used the finite element method to determine a 13% swelling of the extruded liquid jet. Following Tanner's work various publications have established numerically the Newtonian swell as being about 13% for capillary dies and about 19% for slit dies. A comprehensive review of these efforts is presented by Mitsoulis et al. (1984). Apart from the swelling ratios, the numerical results also provide several other aspects of the flow, such as the excess pressure drop over and above the pressure drop that would result if the fluid were in fully developed flow in the die, streamlines, velocity, and pressure and stress distributions inside the die and outside in the exiting stream.

Extrudate swell from annular dies has also received some attention in recent years (Orbey and Dealy, 1984). The main efforts are directed toward experimental investigation of the swelling of polymer melts used in the extrusion blow-molding process. Due to the lack of a reliable constitutive equation that can describe viscoelastic polymer behavior adequately, no numerical simulation has been carried out to study extrudate swell of real polymers from annular dies. The only numerical simulation to date has been performed for a theoretical upper-convected Maxwell fluid for a value of diameter ratio $\kappa = 0.75$ (Crochet and Keunings, 1980).

Since all fluids at very low flow rates behave essentially as Newtonian fluids, it is of interest to examine also extrudate swell of Newtonian fluids in annuli. It is the purpose of this note to provide the corresponding results from numerical calculations on Newtonian fluids for various diameter ratios in flow through annular dies.

Mathematical Formulation and Method of Solution

A schematic diagram of the extrusion through an annular die is given in Figure 1, along with the notation used. Two independent swell ratios can be defined in the case of an annular extrudate (often called "parison" in the plastics industry): the diameter swell, B_1, and the thickness swell, B_2, defined by (Orbey and Dealy, 1984):

$$B_1 = \frac{D_p}{D_o} \tag{1}$$

$$B_2 = \frac{h_p}{h_o} \tag{2}$$

A third swell ratio, the inner diameter swell, B_3, follows from the above definitions:

$$B_3 = \frac{D_p - 2h_p}{D_o - 2h_o} \tag{3}$$

The flow domain is axisymmetric and the governing Navier-Stokes equations are written in cylindrical coordinates as (incompressible fluid, inertialess flow):

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \tag{4}$$

$$0 = -\frac{\partial p}{\partial r} + \frac{\partial \tau_{rr}}{\partial r} + \frac{\tau_{rr}}{r} + \frac{\partial \tau_{rz}}{\partial z} - \frac{\tau_{\theta\theta}}{r} \tag{5}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{\partial \tau_{zz}}{\partial z} \tag{6}$$

The constitutive equation for a Newtonian fluid becomes

$$\overline{\overline{\tau}} = \mu \dot{\overline{\overline{\gamma}}} \tag{7}$$

where μ is a constant viscosity and $\dot{\overline{\overline{\gamma}}}$ is the strain rate tensor with components given by

$$\dot{\gamma}_{rr} = 2\frac{\partial v_r}{\partial r}, \quad \dot{\gamma}_{\theta\theta} = 2\frac{v_r}{r}, \quad \dot{\gamma}_{zz} = 2\frac{\partial v_z}{\partial z}, \quad \dot{\gamma}_{rz} = \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \tag{8}$$

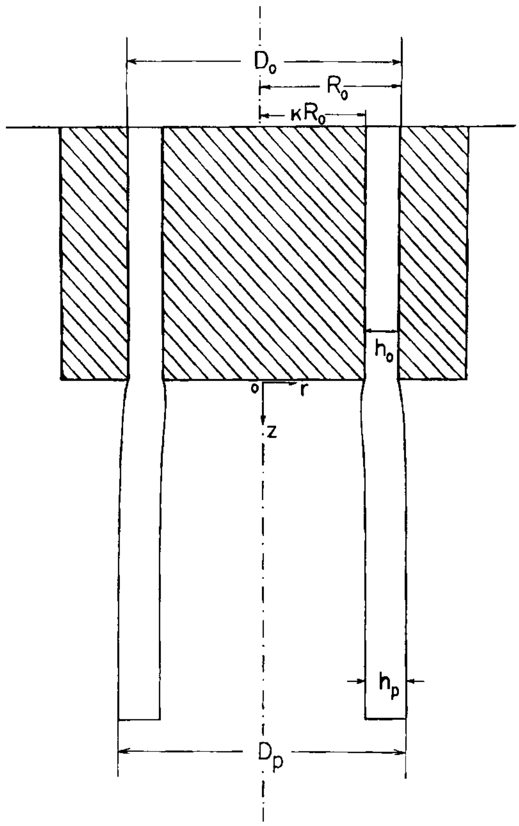

Figure 1. Schematic representation of extrusion through an annular die and notation for the numerical analysis.

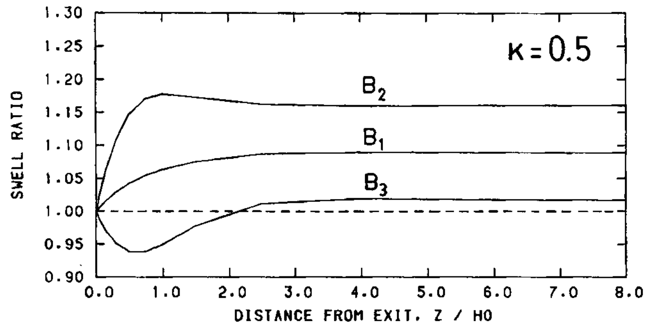

Figure 3. Swell ratios vs. dimensionless distance for Newtonian fluids extruded from an annular die with diameter ratio κ = 0.5.

The appropriate boundary conditions include a fully developed velocity profile upstream, no slip at the annular walls, and $v_r = 0$ at the downstream plane. Also, zero surface forces are specified on the free boundaries. The unknown free surfaces are found iteratively by constructing streamlines according to the method given by Nickell et al. (1974) and applied for annular dies by Crochet and Keunings (1980).

The above differential equations, Eqs. 4–6, are cast in an integral form using the principle of virtual work (Mitsoulis and Vlachopoulos, 1984). The finite element method is then employed for the solution of a discretized set of equations using as primitive variables the velocities v_r, v_z, and pressure p. The domain is subdivided into triangular elements with quadratic variation assumed for the velocities and linear variation assumed for the pressure. The system of linear algebraic equations that results from the finite element discretization is solved using the frontal method of solution (Taylor and Hughes, 1981). Four iterations were found sufficient to locate the free surfaces within less than 0.01% of two successive thickness results divided by the annular gap h_o (i.e., $\Delta h_p/h_o \times 100 \leq 0.01$).

Results and Discussion

The finite element calculations were carried out using the MACVIP finite element program (Mitsoulis et al., 1983) as modified to account for axisymmetric geometries and incorporating the frontal method of solution. The results presented here were obtained for various values of the diameter ratio κ between

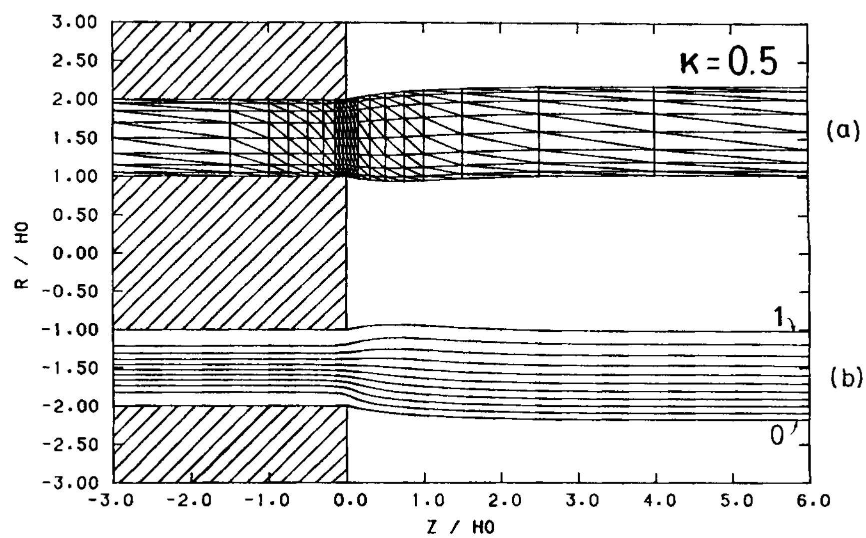

Figure 2. (a) Final finite element grid from the analysis of annular Newtonian extrudate swell (κ = 0.5). (b) Streamlines.

the lower limit of $\kappa = 0$ (capillary die) and the upper limit of $\kappa = 1$ (slit die).

In all cases the finite element grid was extended $8h_0$ upstream and $8h_0$ downstream of the exit. Such entrance and exit lengths are sufficient to justify the imposition of a fully developed velocity profile upstream and $v_r = 0$ downstream. Special care must be taken when selecting the finite element grid. According to Crochet and Keunings (1982), extrudate swell calculations are very sensitive and the values of swell ratios and exit pressures show a significant dependence upon the number of degrees of freedom (and hence the grid density). A dense grid is required near the die exit and near the walls and free surfaces to better capture the drastic changes in the velocities and velocity gradients. The grid used in the present computations was sufficiently dense, consisting of 368 triangular elements, 799 nodes (17 points across) with a total of 1,690 nonzero degrees of freedom (unknowns).

Figure 2 shows the final finite element grid and the streamlines near the exit for $\kappa = 0.5$. The stream function has been obtained a posteriori by integrating the known velocity field (Mitsoulis and Vlachopoulos, 1984). The values have been normalized between 0 (outer streamline) and 1 (inner streamline) with increments of 0.1 in between. A magnified view of the shape of the free surfaces and the behavior of the three swell ratios with distance is shown in Figure 3 for $\kappa = 0.5$.

The results concerning swell ratios and pressure losses are summarized in Table 1. The total pressure drop in the system (ΔP) has been used to evaluate the exit correction (or exit pressure loss) n_{ex}

$$n_{ex} = \frac{\Delta P - \Delta P_o}{2\tau_w} \tag{9}$$

where ΔP_o is the pressure drop for fully developed flow in the annulus and τ_w is the shear stress at the outer wall. Figures 4 and 5 show the swell ratios and exit correction for various κ values, respectively. Points to notice are the highly nonlinear variation of swelling and exit correction with κ. For $\kappa = 0$, the results from capillary calculations are given. The thickness swell (Figure 4c) exhibits an S-shape behavior between the limiting values of 13% (capillary die) and 20% (slit die). The only other comparison that can be made at this point is with the results of Crochet and Keunings (1980) given in Table 1 for $\kappa = 0.75$. The correspondence between these values is good.

Table 1. Effect of Diameter Ratio on Extrudate Swell and Exit Correction for Newtonian Fluids in Annular Flow

Dia. Ratio κ	Dia. Swell B_1%	Inner Dia. Swell B_3%	Thickness Swell B_2%	Exit Correction n_{ex}
0.0	12.81	—	—	0.2430
0.02	12.46	−18.26	13.09	0.2108
0.1	11.87	− 7.06	13.98	0.1891
0.25	10.92	− 1.02	14.90	0.1806
0.5	8.87	1.70	16.05	0.1705
0.75	5.64 (5.5)*	1.69 (1.6)*	17.47 (17)*	0.1624
0.9	2.66	0.88	18.69	0.1581
0.98	0.25	− 0.15	19.63	0.1562
1.0	—	—	19.86	0.1553

*Crochet and Keunings (1980).

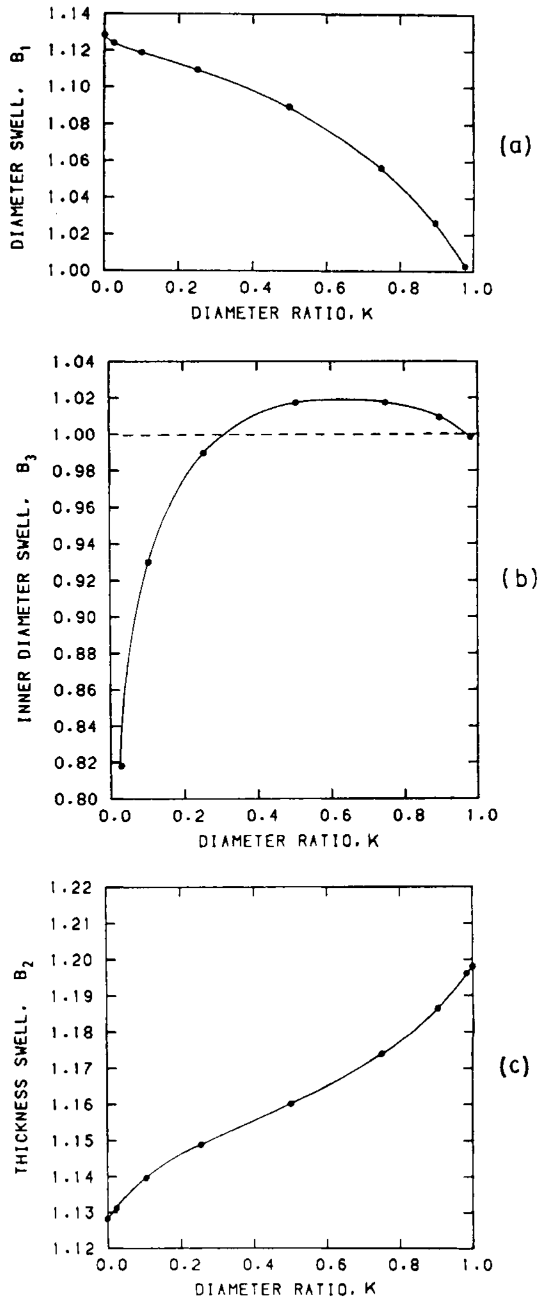

Figure 4. Swell ratios vs. diameter ratio for Newtonian fluids extruded from annular dies: (a) Outer diameter swell. (b) Inner diameter swell. (c) Thickness swell.

Notation

B_1 = (outer) diameter swell
B_2 = thickness swell
B_3 = inner diameter swell
D_o = outer diameter of the annulus, m
D_p = outer diameter of extrudate (parison), m
h_o = annular gap, m
h_p = extrudate (parison) thickness, m
n_{ex} = exit correction
p = pressure, Pa
R_o = outer radius of the annulus, m
r = radial coordinate, m

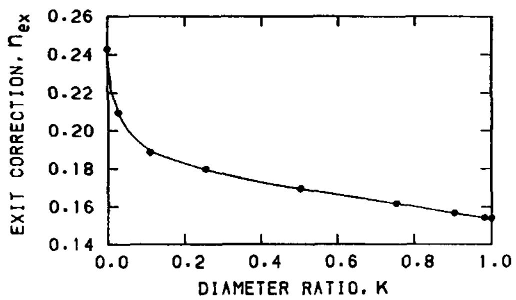

Figure 5. Exit correction vs. diameter ratio for Newtonian fluids extruded from annular dies.

$\overline{V}$ = average velocity, m/s
v_r = radial velocity, m/s
v_z = axial velocity, m/s
z = axial coordinate, m

Greek letters

$\dot{\gamma}$ = strain rate, s^{-1}
ΔP = total pressure drop, Pa
ΔP_o = pressure drop in annulus, Pa
θ = azimuthal coordinate, deg
κ = diameter ratio (inner/outer)
μ = viscosity, Pa · s
τ = stress, Pa
τ_w = wall shear stress, Pa

Subscripts

ex = exit
i = inner
o = outer, annulus
r = r-coordinate
w = wall
z = z-coordinate

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